

Correlated Momentum distributions in nuclear systems

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- Motivation
- Single particle spectral functions at zero and finite Temperature
- Single-particle properties
- Entropy and Free energy
- Thermodynamical properties. Liquid-gas coexistence.
- Conclusions and perspectives

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The equation of state (**EOS**) of asymmetric matter (different number of protons and neutrons) is a necessary ingredient in the description of astrophysical environments such as supernova explosions or the structure of neutron stars.

Recent data concerning nuclei far from the stability valley has revitalized the interest for asymmetric systems.

Heavy ion collisions.

The EOS of asymmetric nuclear matter in a wide range of densities and temperatures is one of the challenging open problems in nuclear physics.

It is a very old many-body problem and still has not been fully solved. We have two problems : the NN interaction and the many-body problem.

Here we will assume that the NN interaction is realistic and we will concentrate in the many-body problem.

The microscopic study of the single particle properties in nuclear systems requires a rigorous treatment of the nucleon-nucleon (NN) correlations.

- Strong short range repulsion and tensor components, in realistic interactions to fit NN scattering data $\Rightarrow \Rightarrow$ Important modifications of the nuclear wave function.
- Simple Hartree-Fock for nuclear matter at the empirical saturation density using such realistic NN interactions provides positive energies rather than the empirical -16 MeV per nucleon.
- The effects of correlations appear also in the single-particle properties:
 - Partial occupation of the single particle states which would be fully occupied in a mean field description and a wide distribution in energy of the single-particle strength. Evidencies from (e,e'p) experiments.

The Single particle propagator a good tool to study single particle properties

Not necessary to know all the details of the system (the full many-body wave function) but just what happens when we add or remove a particle to the system.

It gives access to all single particle properties as :

- momentum distributions
- self-energy (Optical potential)
- effective masses
- spectral functions

Also permits to calculate the expectation value of a very special two-body operator: the Hamiltonian in the ground state.

Recently, enormous progress has been achieved in the calculation of the single-particle propagator:

Self-consistent Green's function (**SCGF**) and Correlated Basis Function (**CBF**).

Single particle propagator

Zero temperature

$$ig(\mathbf{k}t, \mathbf{k}'t') = \langle \Psi_0 | T [a_{\mathbf{k}}(t) a_{\mathbf{k}'}^\dagger(t')] | \Psi_0 \rangle$$

Heisenberg picture

$$a_{\mathbf{k}}(t) = e^{it\hat{H}} a_{\mathbf{k}} e^{-it\hat{H}}$$

T is the time ordering operator

Finite temperature

$$ig(\mathbf{k}t, \mathbf{k}'t') = Tr \left\{ \rho T [a_{\mathbf{k}}(t) a_{\mathbf{k}'}^\dagger(t')] \right\}$$

The trace is to be taken over all energy eigenstates and all particle number eigenstates of the many-body system

$$\rho = \frac{1}{Z} e^{-\beta(\hat{H} - \mu\hat{N})}$$

$$Z = Tr e^{-\beta(\hat{H} - \mu\hat{N})}$$

✓ Z is the grand partition function

FT+ closure $\Rightarrow$ Lehmann representation

$$g(k, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega' \frac{A^>(k, \omega')}{\omega - \omega' + i\eta} + \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega' \frac{A^<(k, \omega')}{\omega - \omega' - i\eta}$$

$$A^<(k, \omega) = 2\pi \sum_{n,m} \frac{e^{-\beta(E_m - \mu N_m)}}{Z} \left| \langle \Psi_n | a_{\mathbf{k}} | \Psi_m \rangle \right|^2 \delta(\omega - (E_m - E_n))$$

$$A^>(k, \omega) = 2\pi \sum_{m,n} \frac{e^{-\beta(E_m - \mu N_m)}}{Z} \left| \langle \Psi_n | a_{\mathbf{k}}^\dagger | \Psi_m \rangle \right|^2 \delta(\omega - (E_m - E_0))$$

The summation runs over all energy eigenstates and all particle number eigenstates

The spectral function

$$A(k, \omega) = A^<(k, \omega) + A^>(k, \omega)$$

with $A^>(k, \omega) = e^{\beta(\omega - \mu)} A^<(k, \omega)$

therefore $A^<(k, \omega) = A(k, \omega) f(\omega)$

where $f(\omega) = \left\{ e^{\beta(\omega - \mu)} + 1 \right\}^{-1}$

Is the Fermi function

and $A^>(k, \omega) = A(k, \omega)(1 - f(\omega))$

Momentum distribution

T=0 MeV

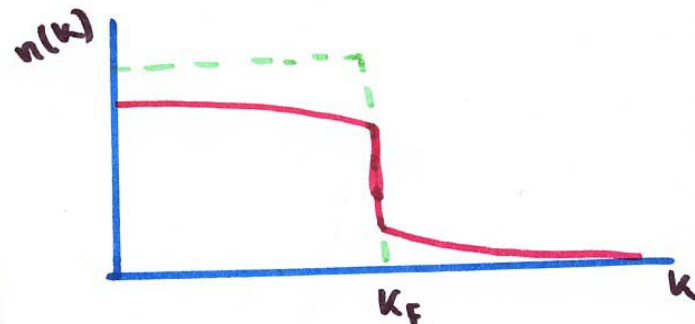
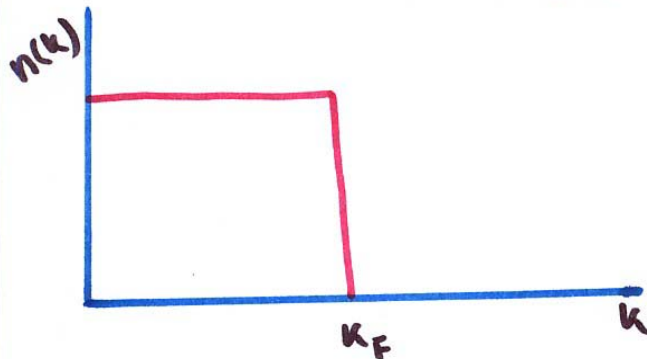
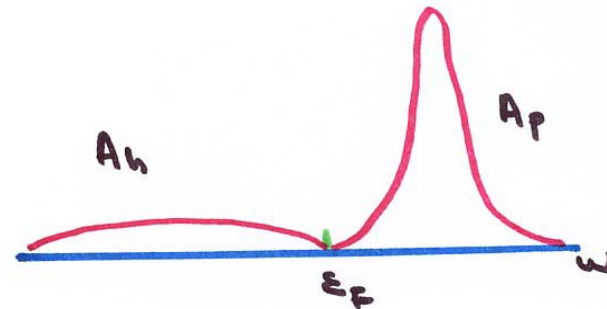
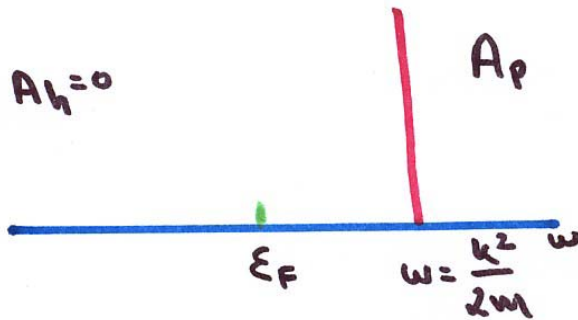
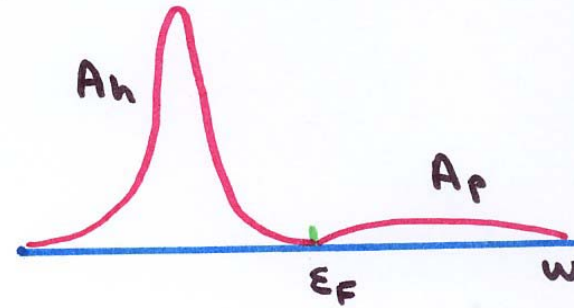
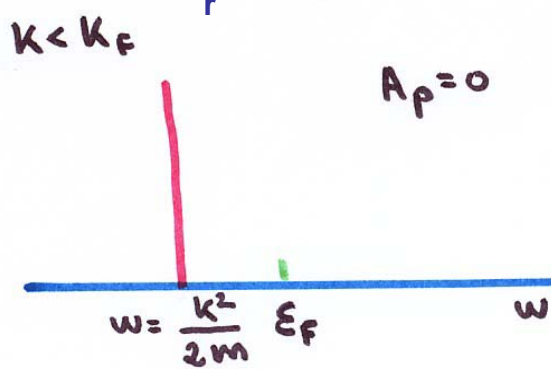
$$n(k, T = 0) = \frac{1}{2\pi} \int_{-\infty}^{\epsilon_F} A_h(k, \omega) d\omega$$

Finite T

$$n(k, T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A^<(k, \omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(k, \omega) f(\omega) d\omega$$

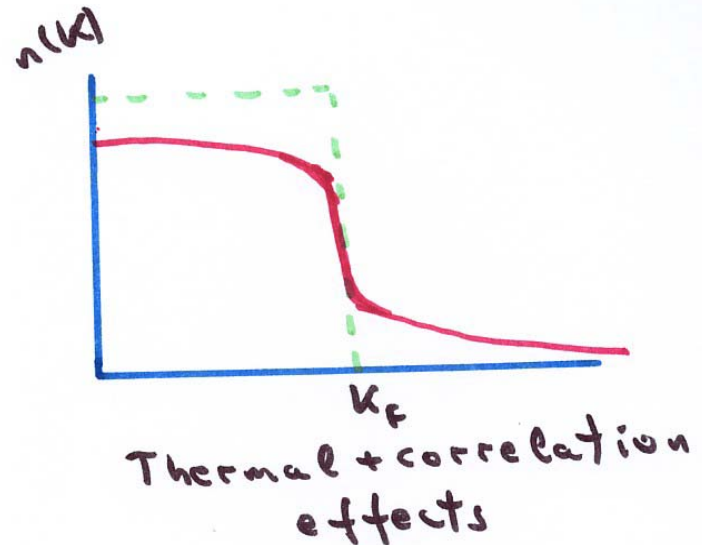
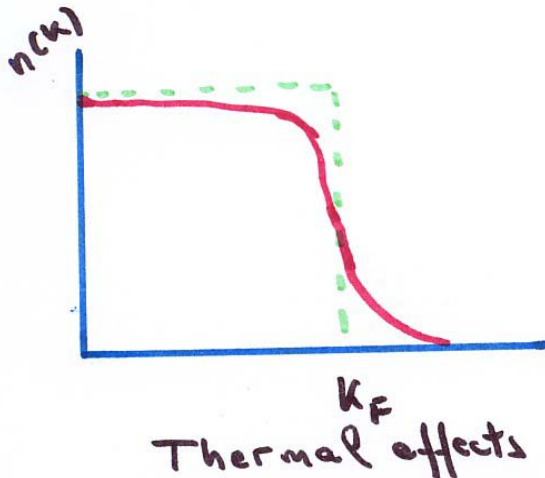
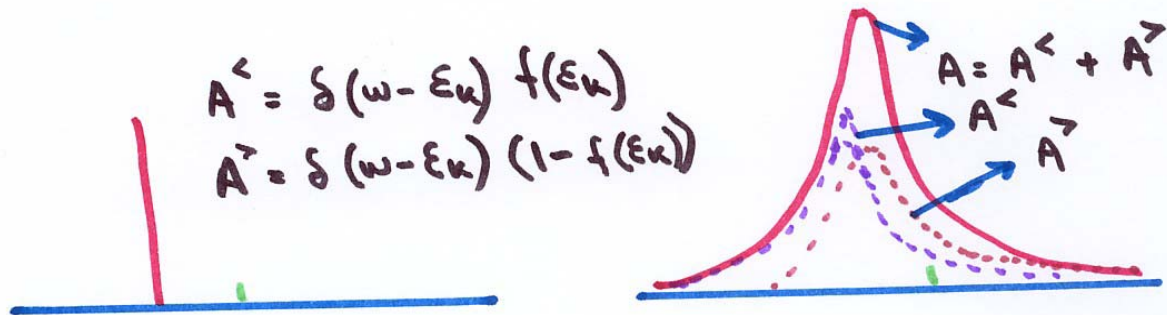
Spectral functions at zero temperature

Free system $\Rightarrow$ Interactions $\Rightarrow$ Correlated system

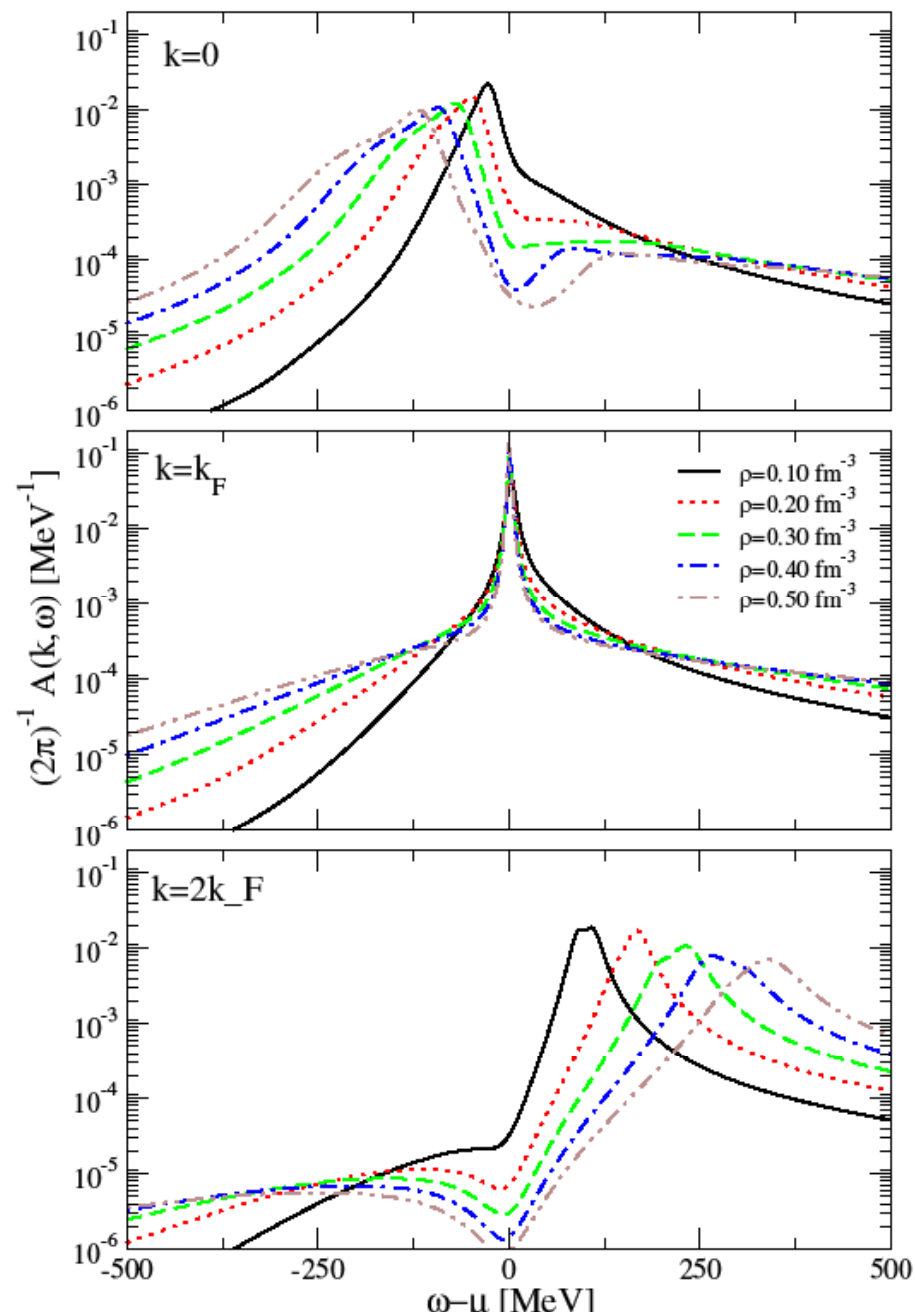


Spectral functions at finite Temperature

Free system $\Rightarrow$ Interactions $\Rightarrow$ Correlated system



Symmetric Nuclear Matter, $T=10$ MeV

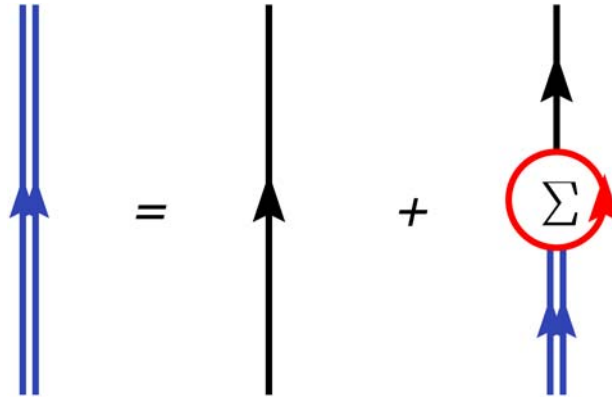


Tails extend to the high energy range.

Quasi-particle peak shifting with density.

Peaks broaden with density.

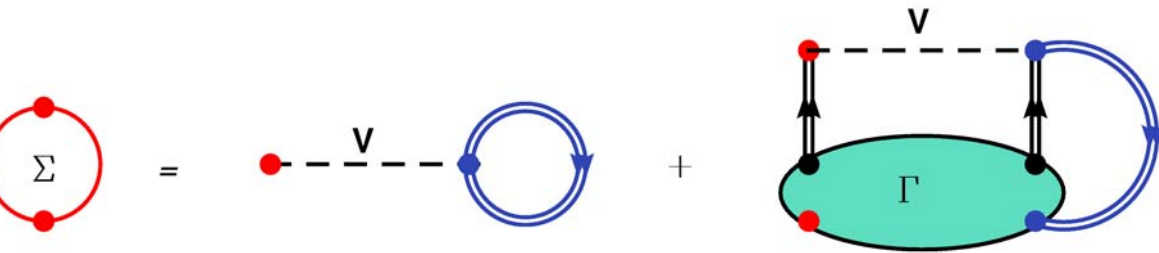
Dyson equation



$$A(k, \omega) = \frac{-2\text{Im}\Sigma(k, \omega + i\eta)}{\left[\omega - \frac{k^2}{2m} - \text{Re}\Sigma(k, \omega)\right]^2 + [\text{Im}\Sigma(k, \omega + i\eta)]^2}$$

$$g(k, \omega) = \int_{-\infty}^{\infty} d\omega' \left[\frac{A(k, \omega') f(\omega')}{\omega - \omega' - i\eta} + \frac{A(k, \omega') (1 - f(\omega'))}{\omega - \omega' + i\eta} \right]$$

How to calculate the self-energy



The self-energy accounts for the interactions of a particle with the particles in the medium.

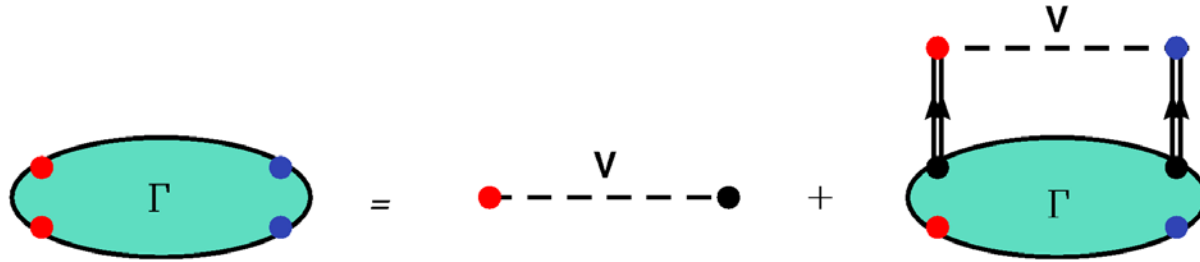
We consider the irreducible self-energy. The repetitions of this block are generated by the Dyson equation.

The first contribution corresponds to a generalized HF, weighted with $n(k)$

The second term contains the renormalized interaction, which is calculated in the ladder approximation by propagating particles and holes. The ladder is the minimum approximation that makes sense to treat short-range correlations.

It is a complex quantity, one calculates its imaginary part and after the real part is calculated by dispersion relation.

The interaction in the medium



$$\langle q | \Gamma | q' \rangle = \langle q | V | q' \rangle + \int d\mathbf{k}' \langle q | V | k' \rangle \bar{g}_{II}(P, \Omega, k') \langle k' | \Gamma(P, \Omega) | q' \rangle$$

$$\Gamma_I(P, \Omega, q) = \int \int d\omega d\omega' \left[\frac{A^<(k_1, \omega) A^<(k_2, \omega')}{\Omega - \omega - \omega' - i\eta} - \frac{A^>(k_1, \omega) A^>(k_2, \omega')}{\Omega - \omega - \omega' + i\eta} \right]$$

How to calculate the energy

Koltun sum-rule

$$\frac{E}{A} = \frac{deg}{\rho} \sum_{\tau} \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{2} \left(\frac{k^2}{2m} + \omega \right) A_{\tau}(k, \omega) f_{\tau}(\omega)$$

The BHF approach $\Rightarrow A_{\tau}(k, \omega) = \delta(\omega - \epsilon_{\tau}^{BHF}(k))$

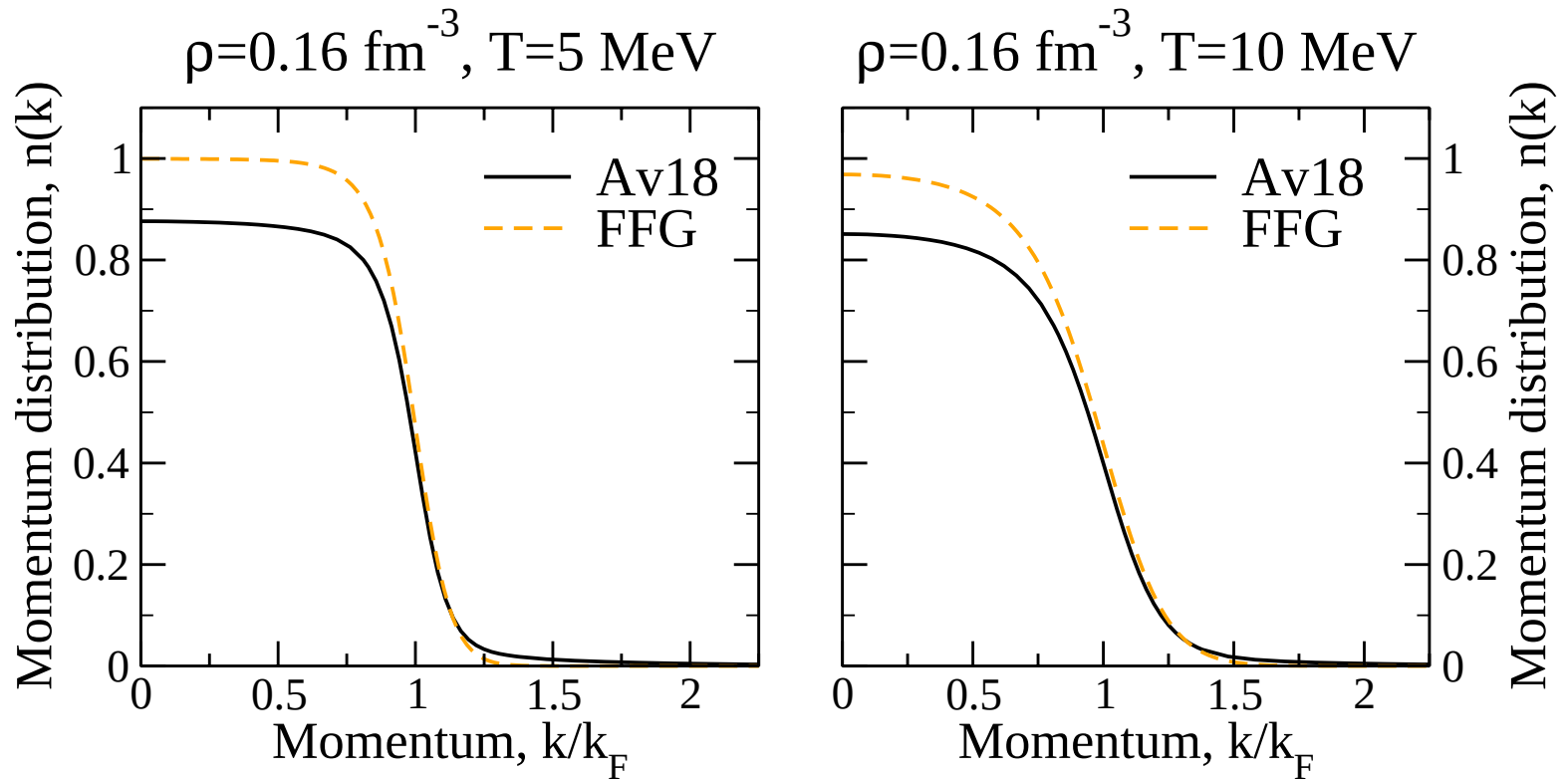
$\epsilon_{\tau}^{BHF}(k)$ is the BHF quasi-particle energy

Does not include propagation of holes

Momentum distributions for symmetric nuclear matter

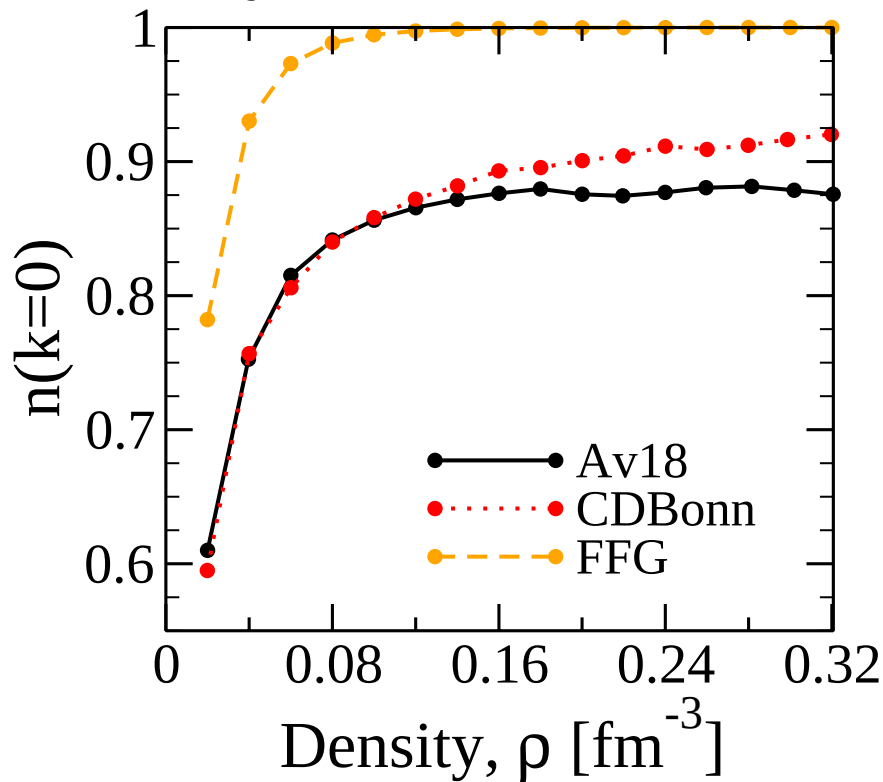
At $T = 5$ MeV, for FFG $k < k_F$ 86 per cent of the particles! and 73 per cent at $T = 10$ MeV. In the correlated case, at $T = 5$ MeV, for FFG $k < k_F$ 75 per cent and

66 p

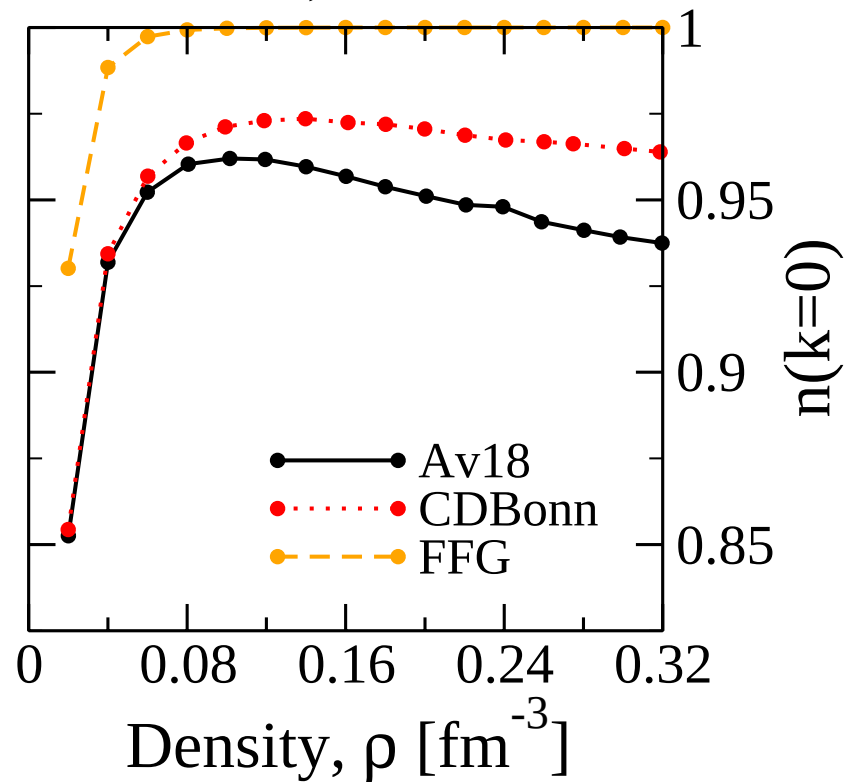


Density dependence of the occupation of lowest momentum state at $T=5$ MeV .

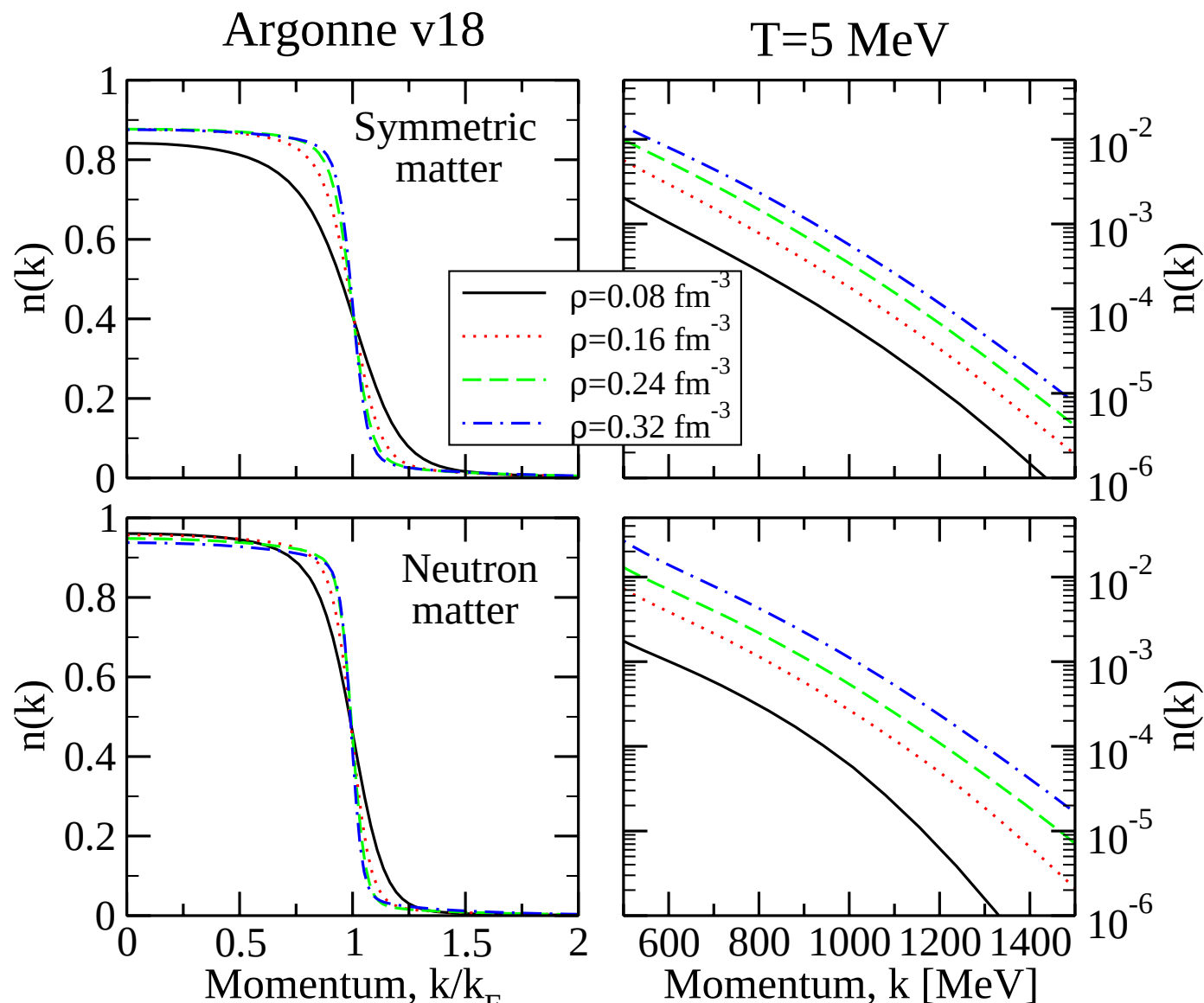
Symmetric, $T=5$ MeV



Neutron, $T=5$ MeV



Momentum distributions of symmetric and neutron matter at T=5 MeV



Relation of the momentum distribution and the derivative of the real part time ordered self-energy at the quasi-particle energy.

$$n(k) = \frac{1}{1 - \partial_{\omega} \text{Re} \Sigma_{\downarrow}(k, \omega) \big|_{\omega=\epsilon(k)}}, k < k_F$$

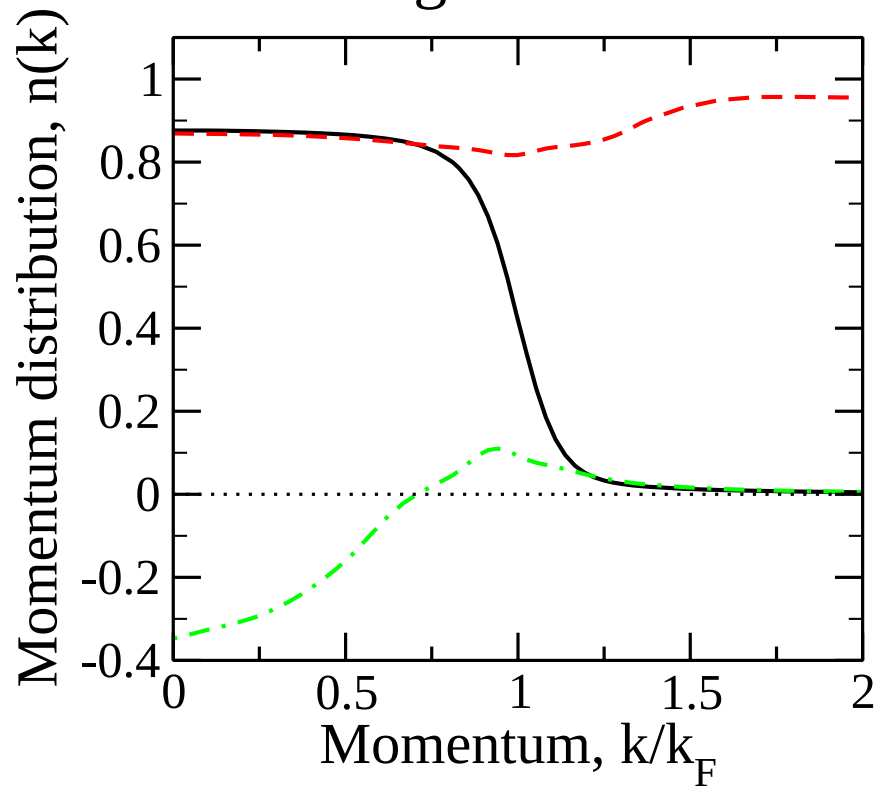
$$n(k) = 1 - \frac{1}{1 - \partial_{\omega} \text{Re} \Sigma_{\uparrow}(k, \omega) \big|_{\omega=\epsilon(k)}}, k > k_F$$

The self-energy down has contributions from 2p1h self-energy diagrams

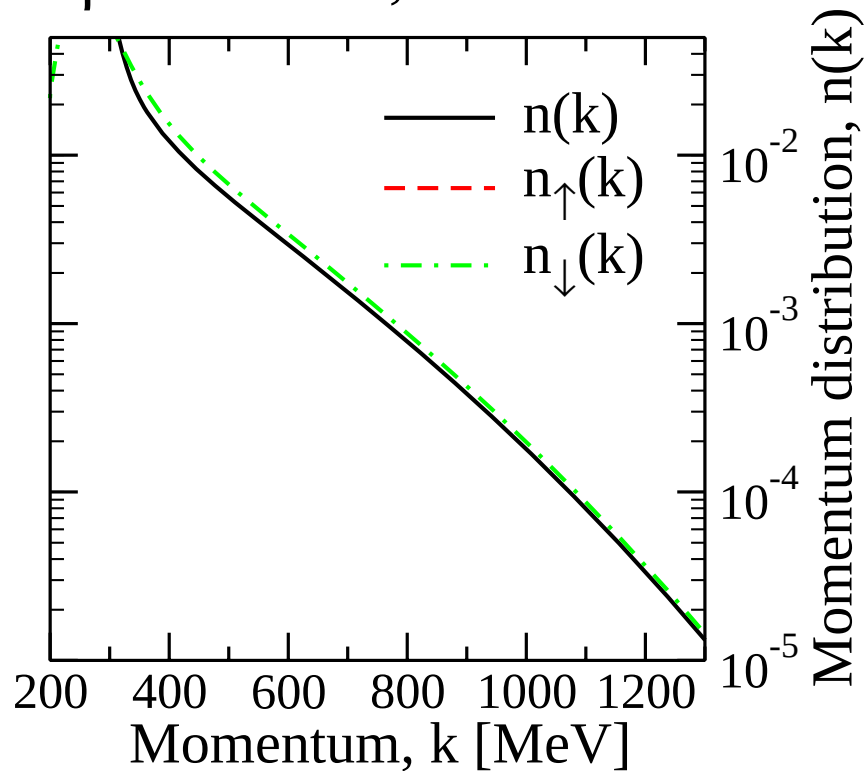
The self-energy up has contributions from 2h1p self-energy diagrams

Momentum distributions obtained from the derivatives of the self-energy

Argonne v18

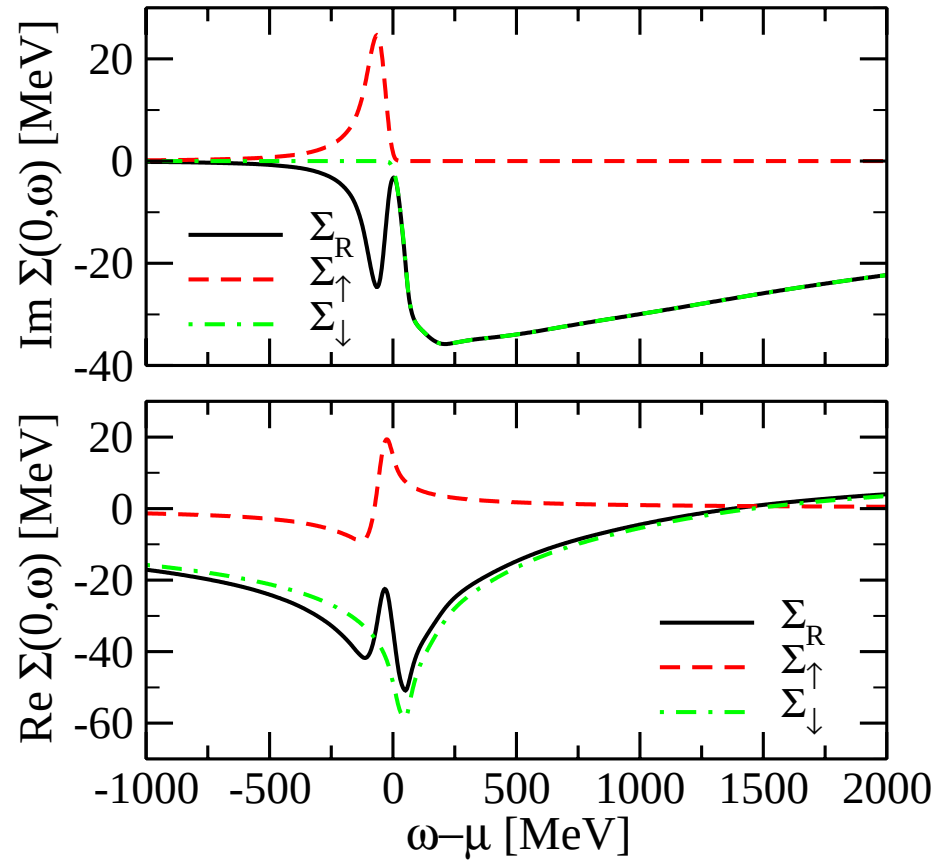


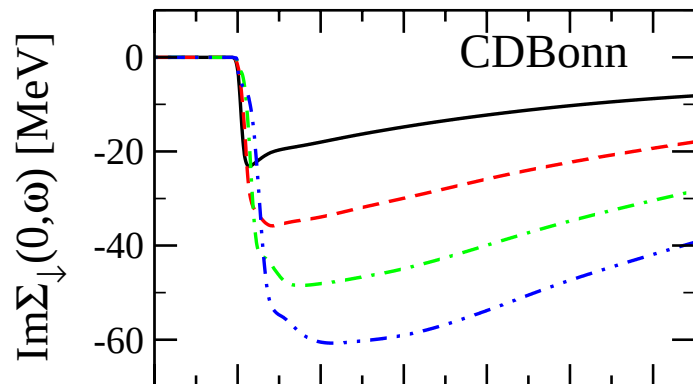
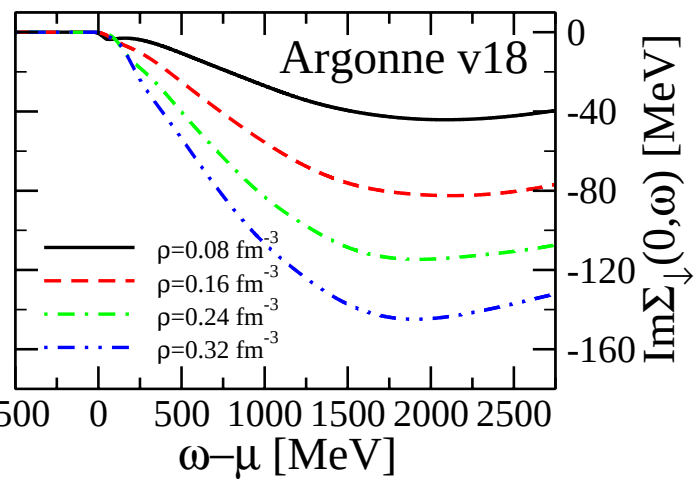
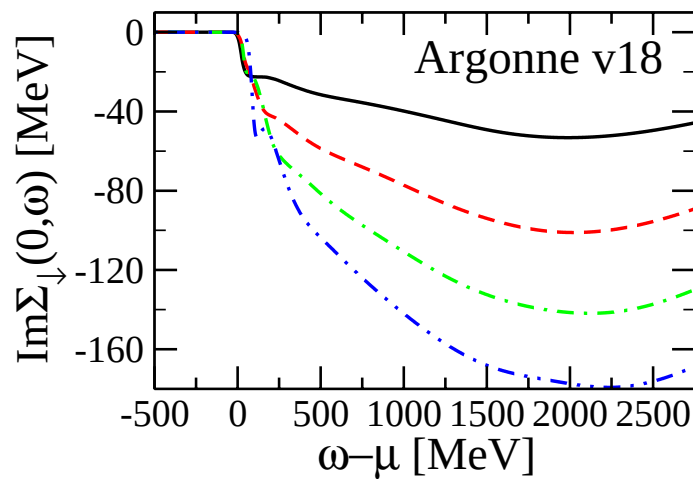
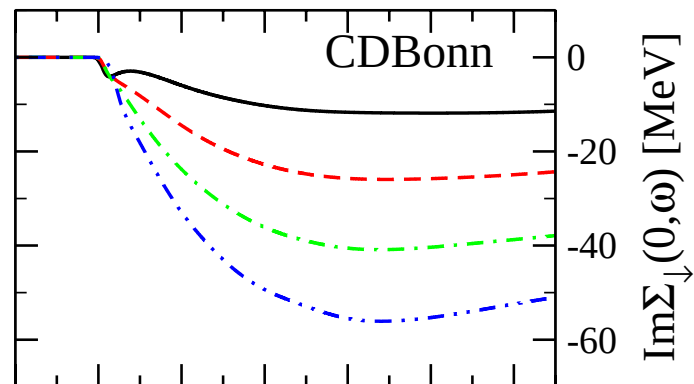
$\rho = 0.16 \text{ fm}^{-3}$, $T = 5 \text{ MeV}$



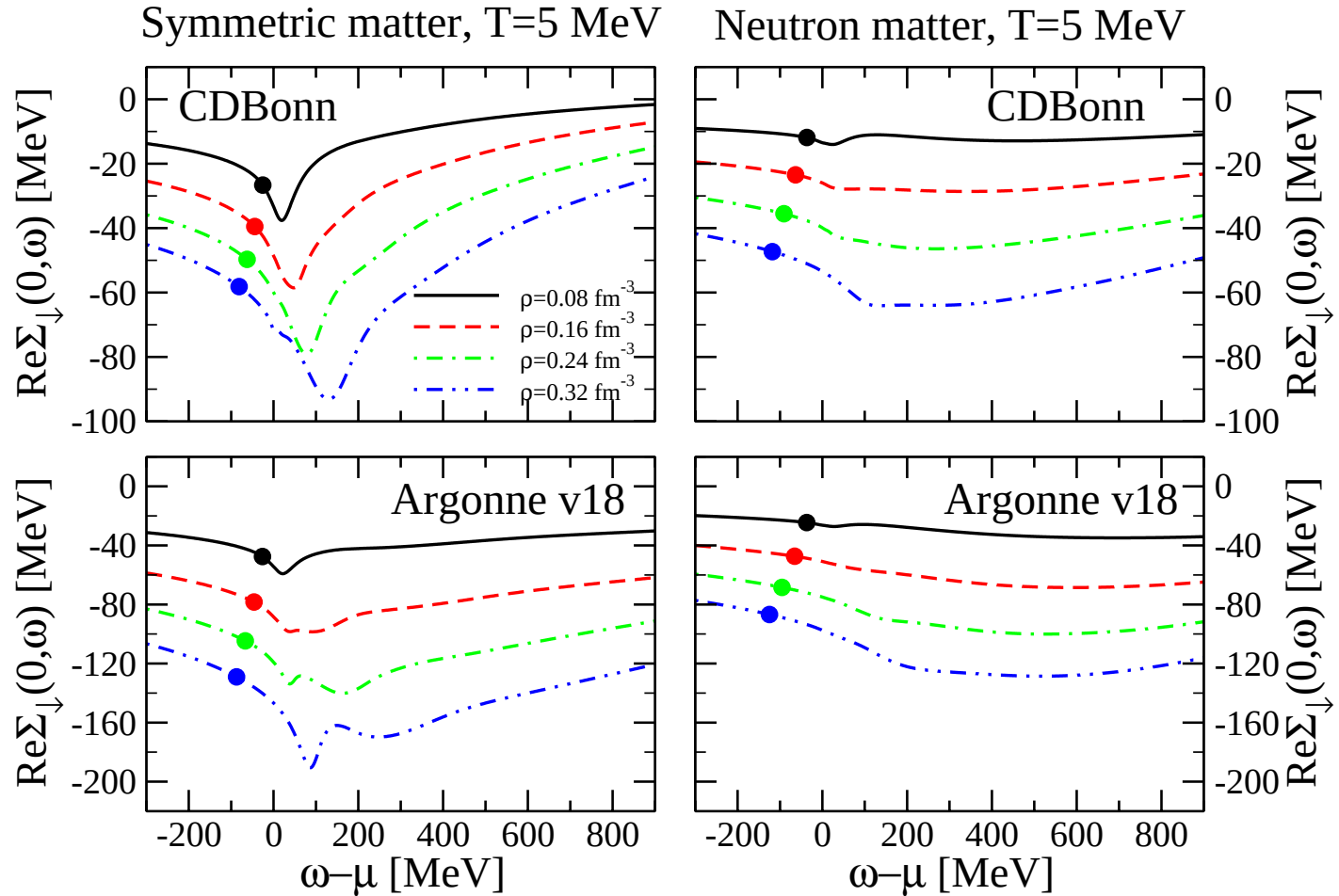
Different components of the imaginary and real parts of the self-energy

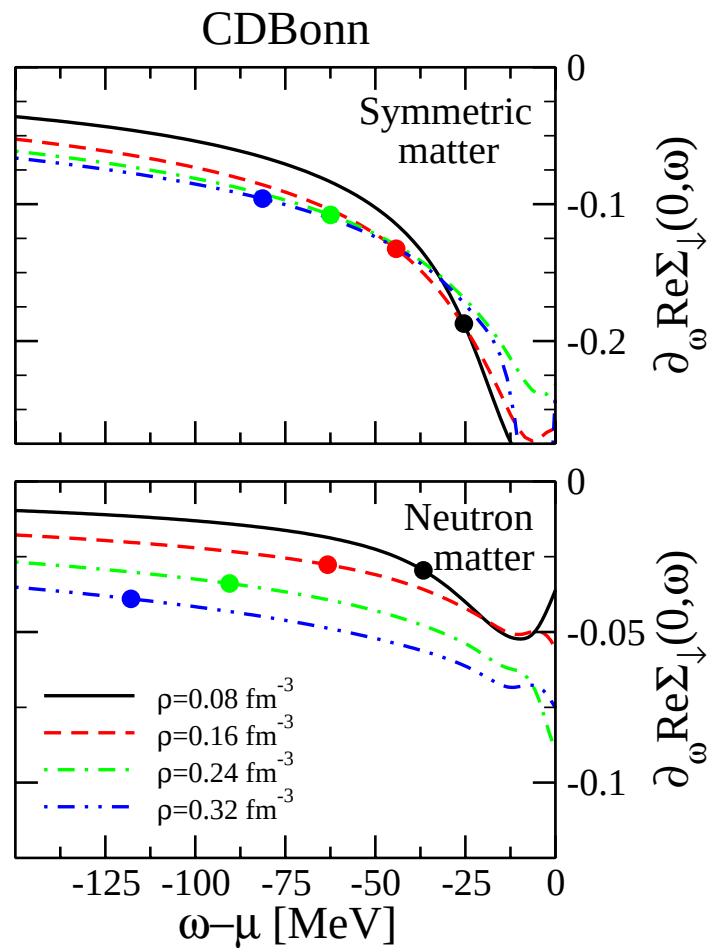
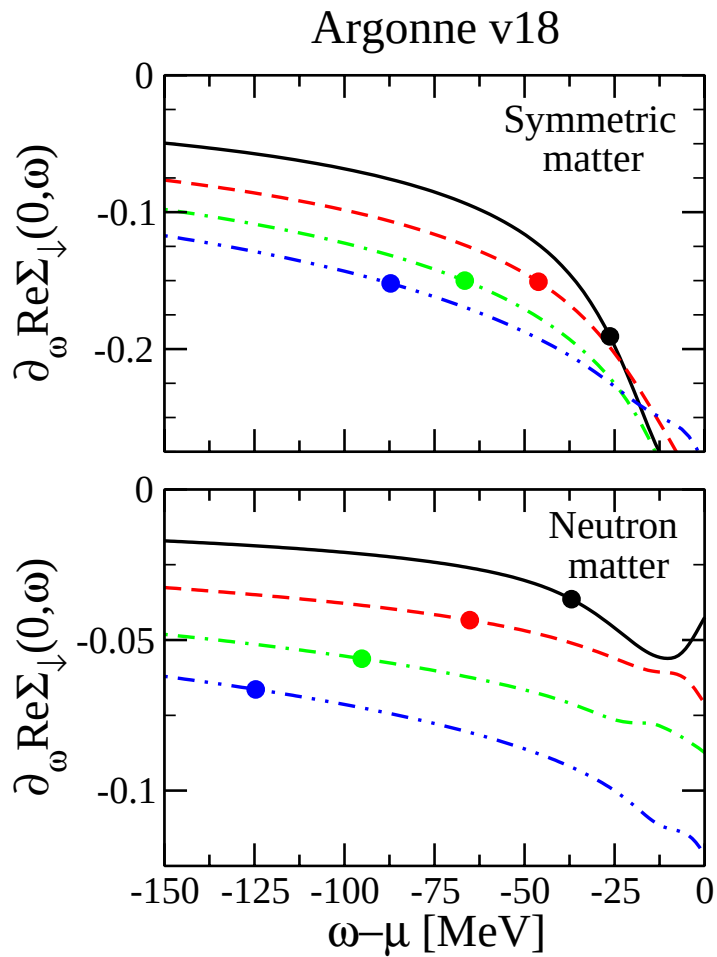
CDBonn, $\rho=0.16 \text{ fm}^{-3}$, $T=5 \text{ MeV}$



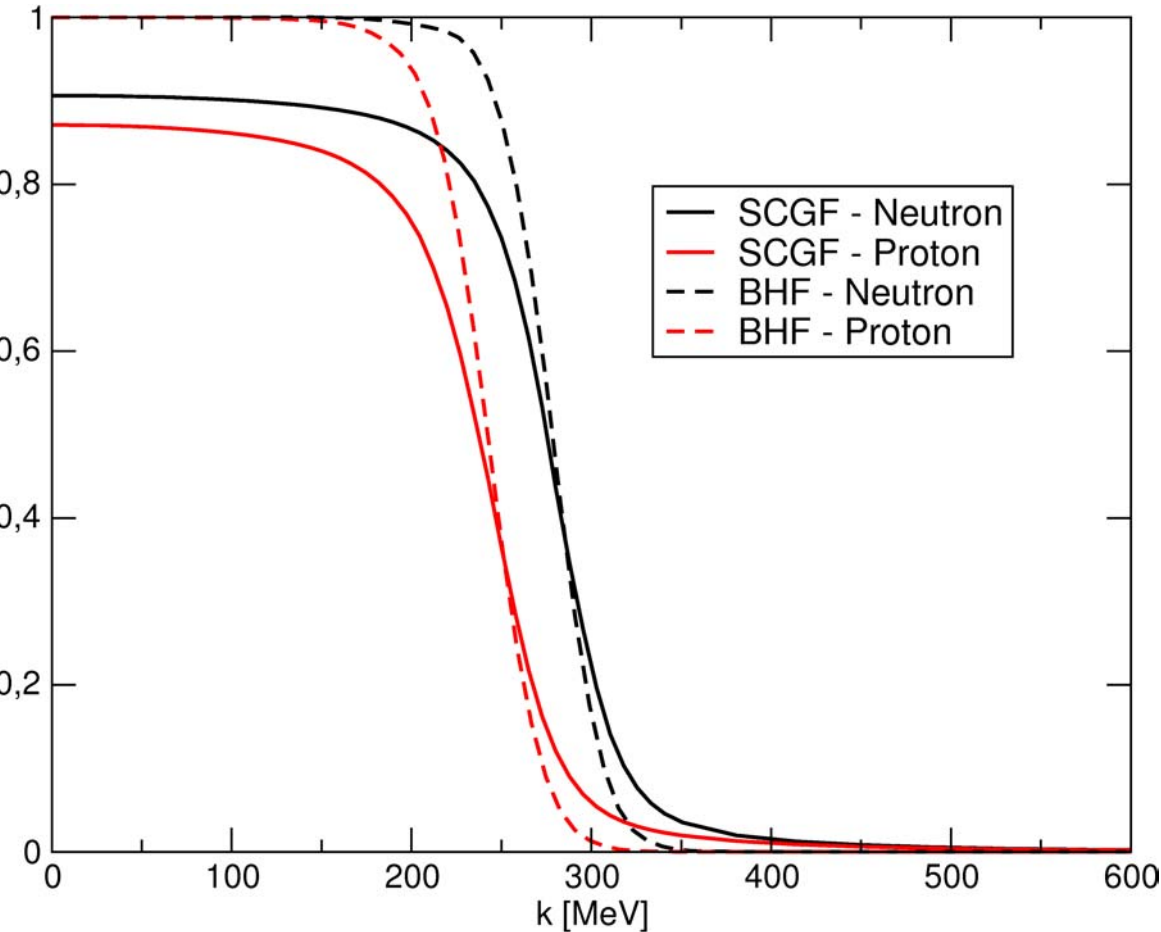
Symmetric matter, $T=5$ MeVNeutron matter, $T=5$ MeV

The circles represent the position of the quasi-particle energy





Proton and neutron momentum distributions $\alpha=0.2$, $\rho=0.16 \text{ fm}^{-3}$

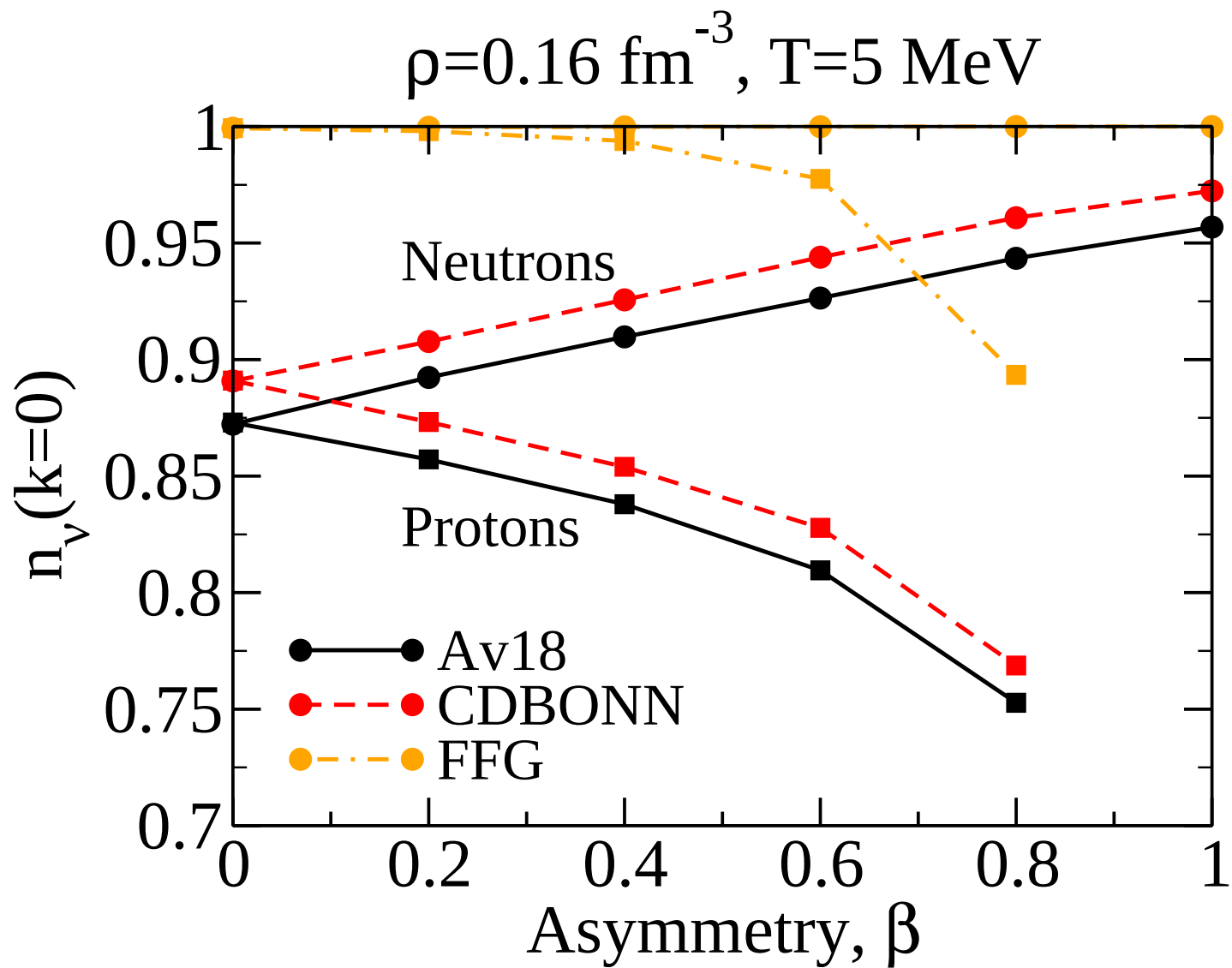


✓The BHF $n(k)$ do not contain correlation effects and very similar to a normal thermal Fermi distribution.

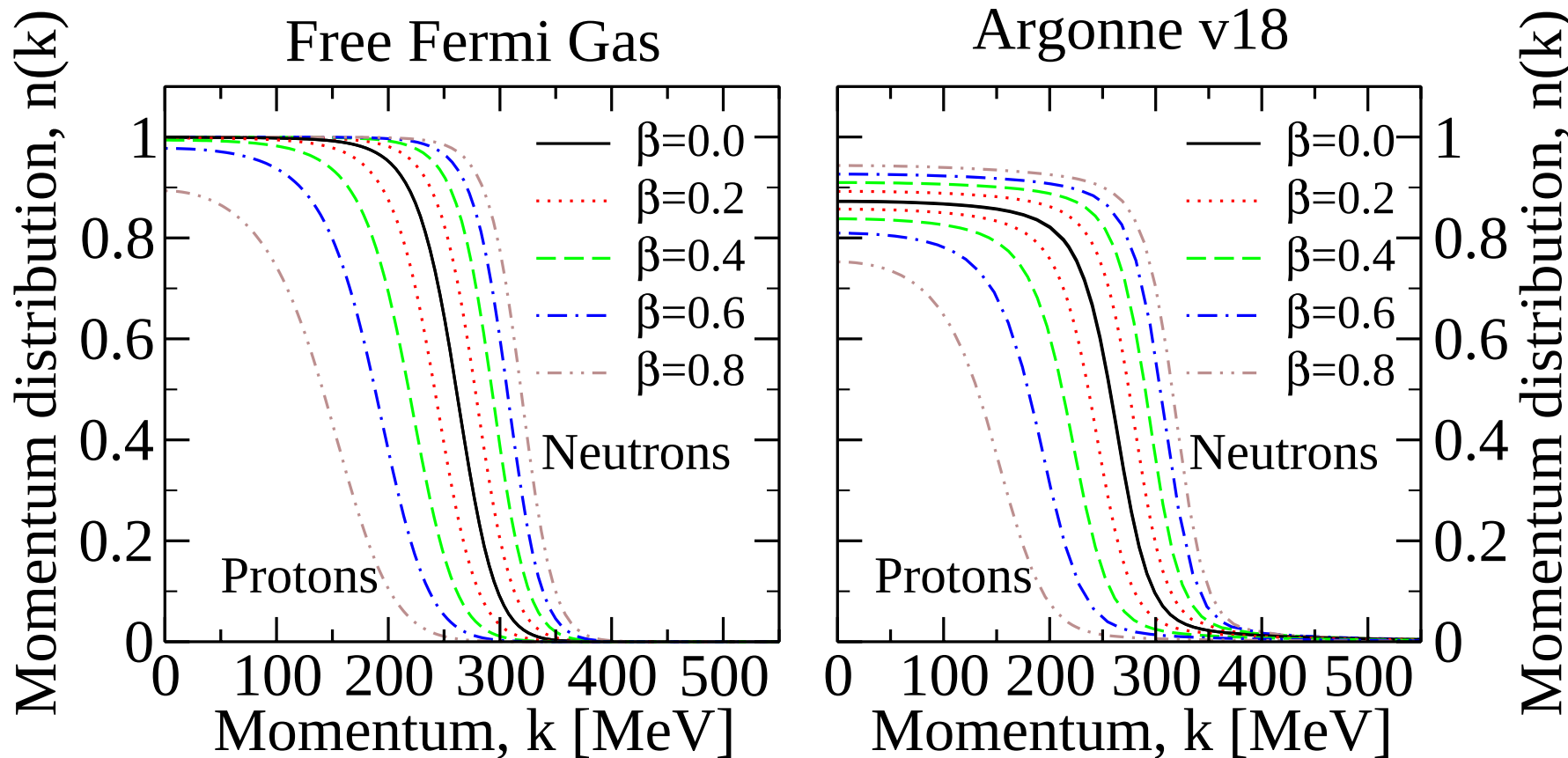
✓The SCGF $n(k)$ contain thermal and correlation effects.

✓Depletion at low momenta and larger occupation than the BHF $n(k)$ at larger momenta.

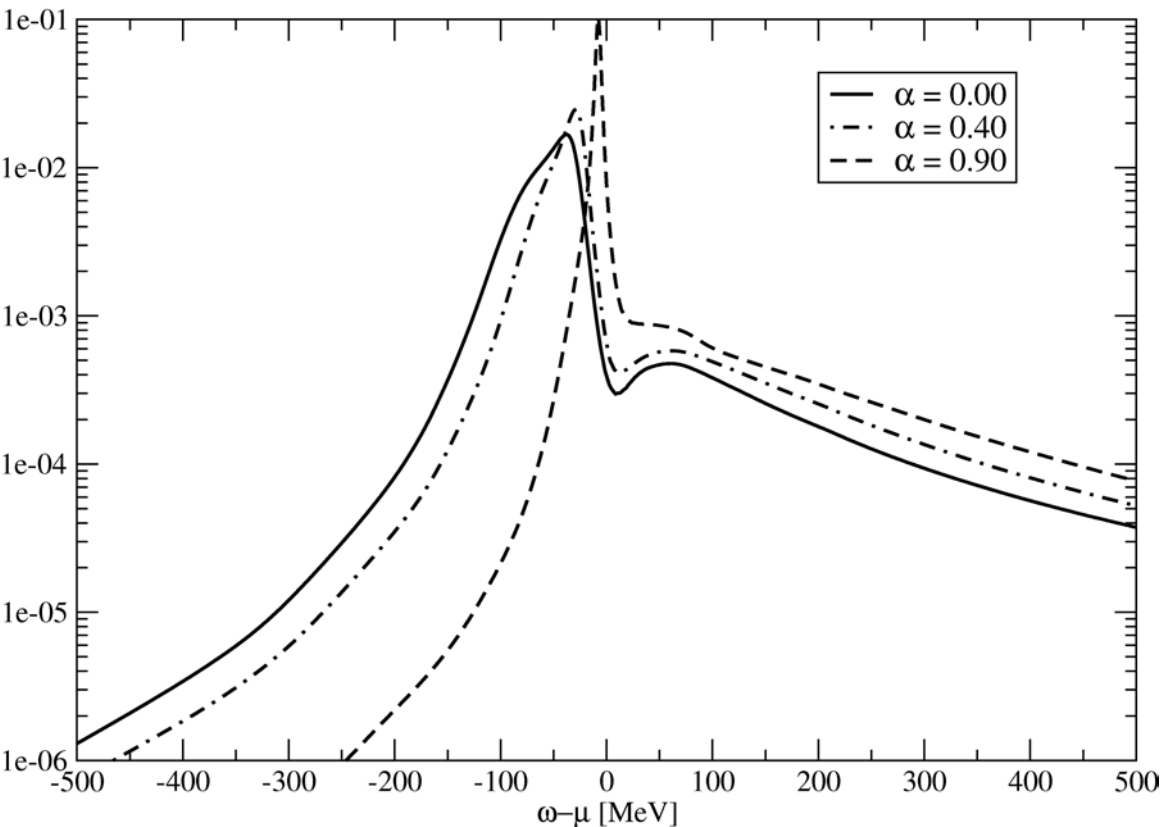
✓The proton depletion is larger than the neutron depletion. Relevant for $(e,e'p)$.



Neutron and proton momentum distributions for different asymmetries



K=0 MeV proton spectral function for different asymmetries



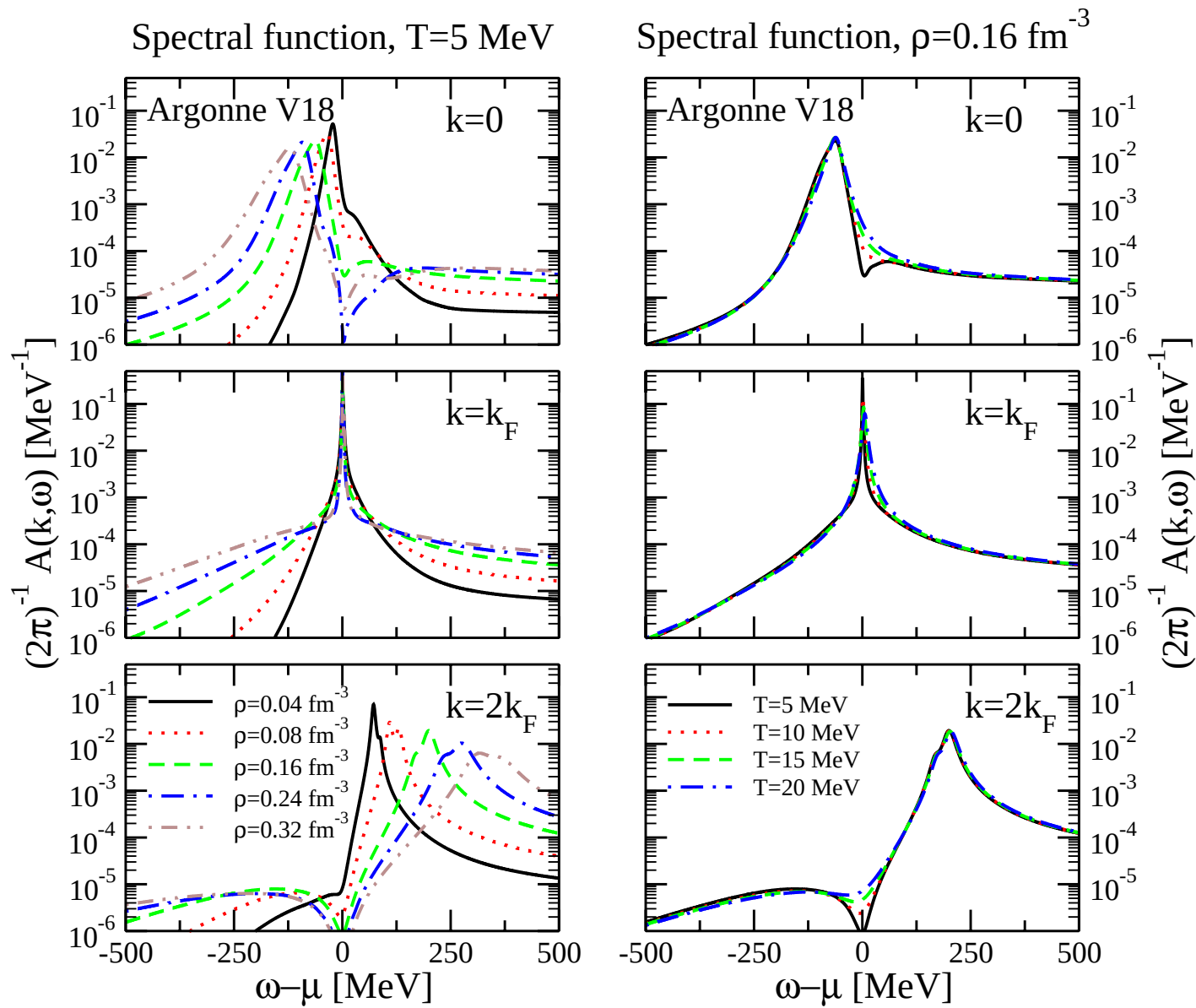
✓ $\alpha \rightarrow 1$, $k_F^p \rightarrow 0$ MeV, the quasi-particle peak gets narrower and higher.

✓ The spectral function at positive energies is larger with increasing asymmetry.

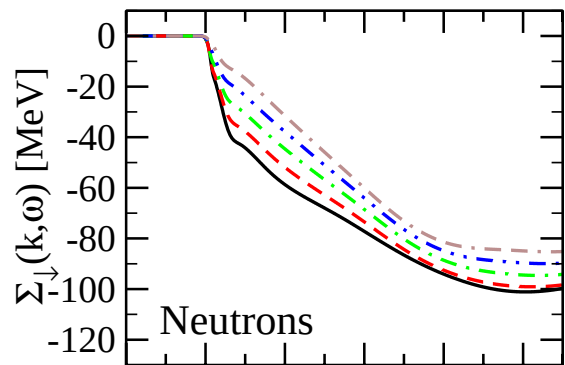
✓ Tails extend to the high-energy range.

✓ Peak broadens with density

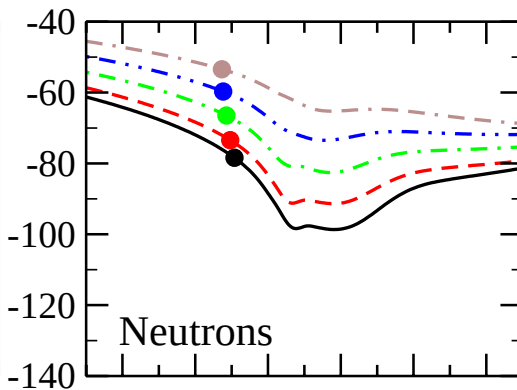
Density and temperature dependence of the spectral function for neutron matter



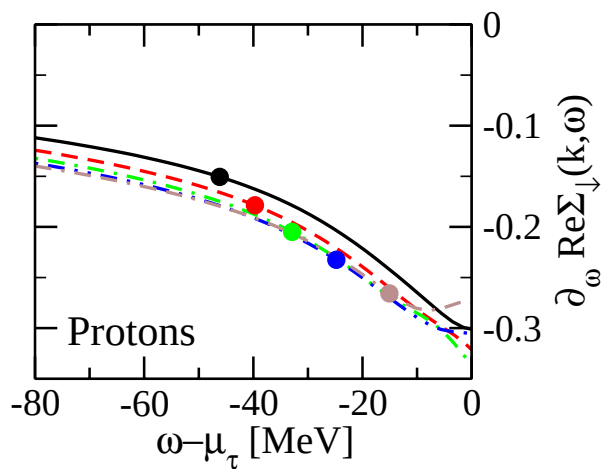
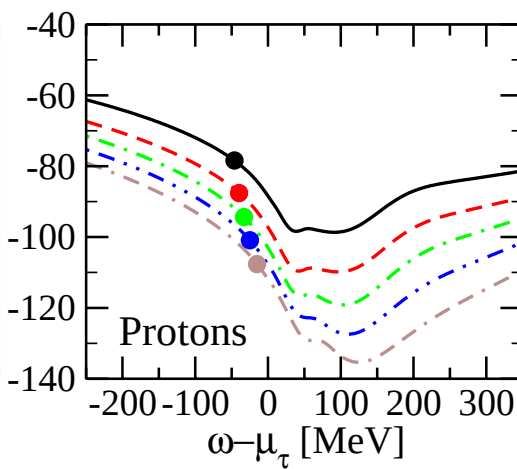
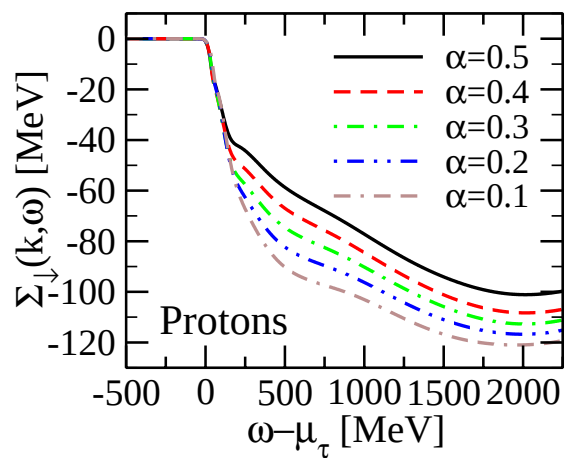
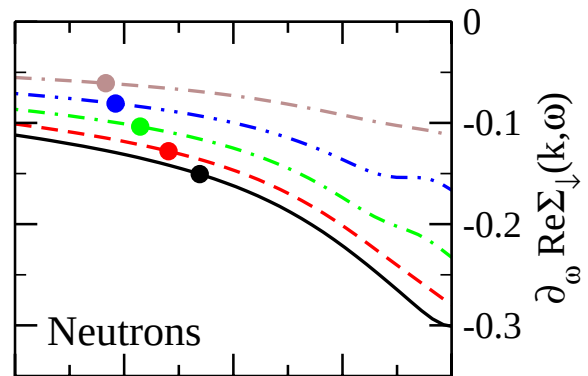
Imaginary part



Real part



Partial derivative

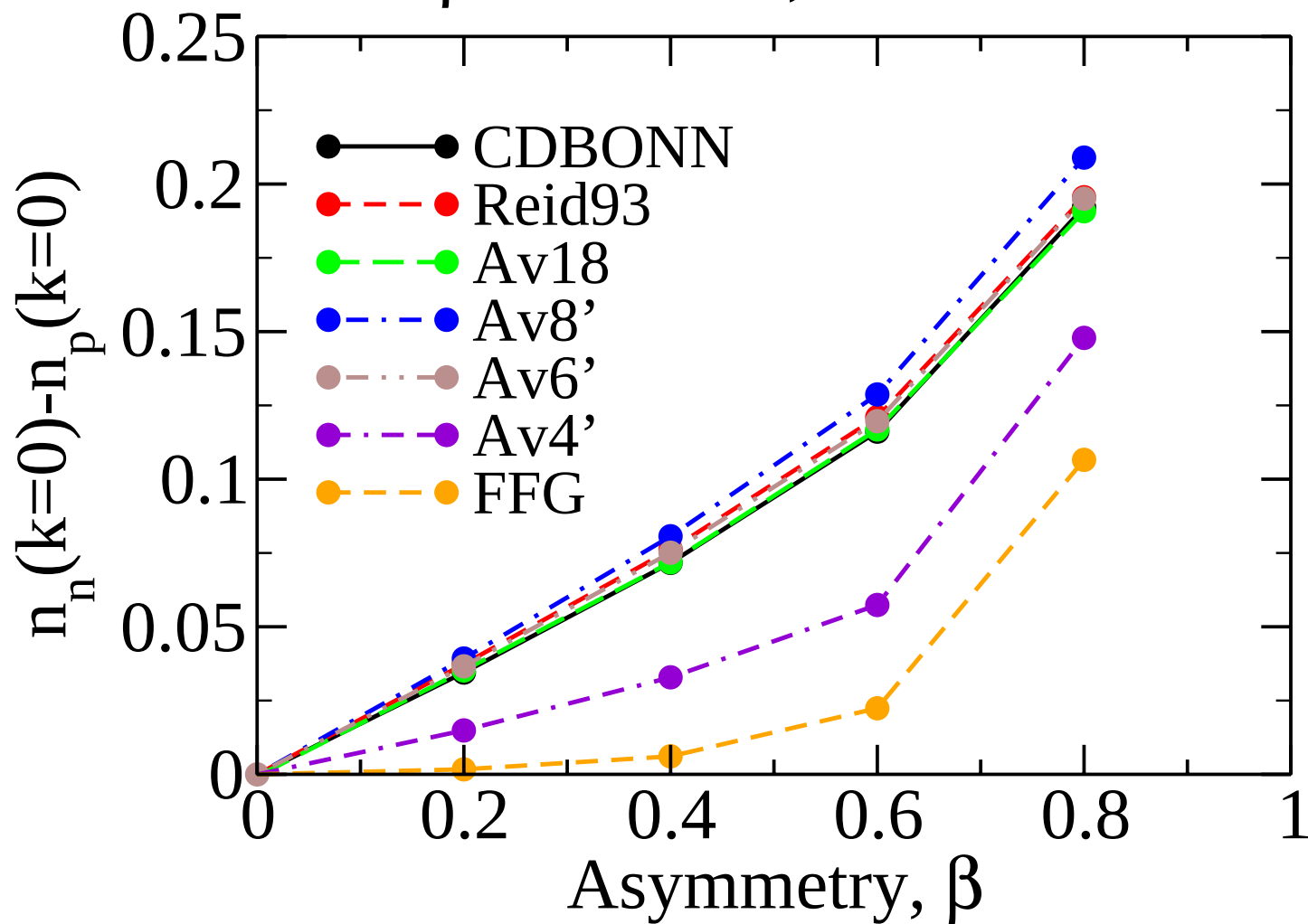


n(k=0) for nuclear and neutron matter,

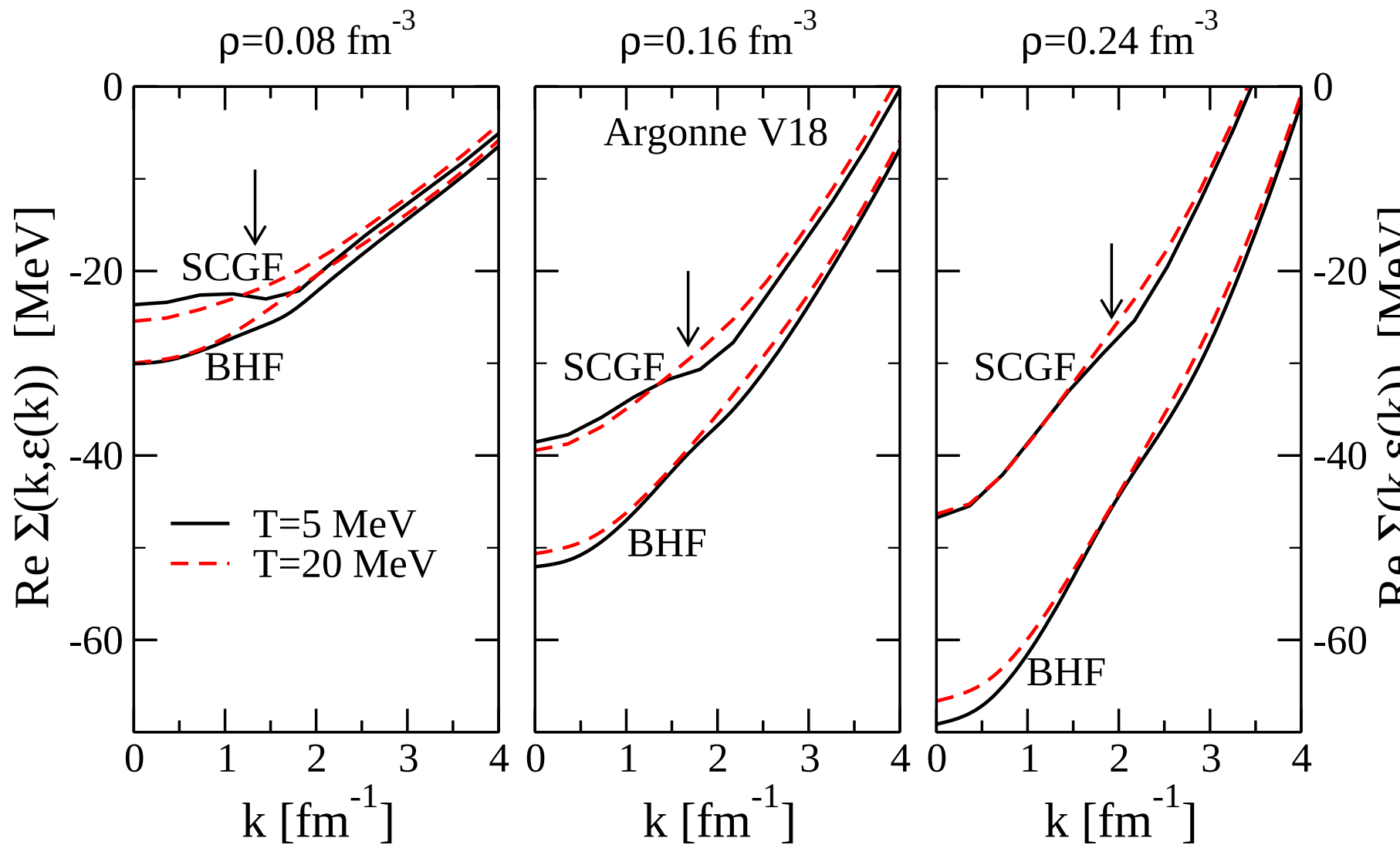
$$\rho = 0.16 \text{ fm}^{-3} \text{ and } T = 5 \text{ MeV}$$

Interaction	Symmetric	Neutron
CDBONN	0.891	0.972
Reid93	0.872	0.962
Argonne v18	0.872	0.957
Argonne v8'	0.863	0.956
Argonne v6'	0.879	0.964
Argonne v4'	0.946	0.971
FFG	0.9993	0.999991

$\rho=0.16 \text{ fm}^{-3}$, $T=5 \text{ MeV}$

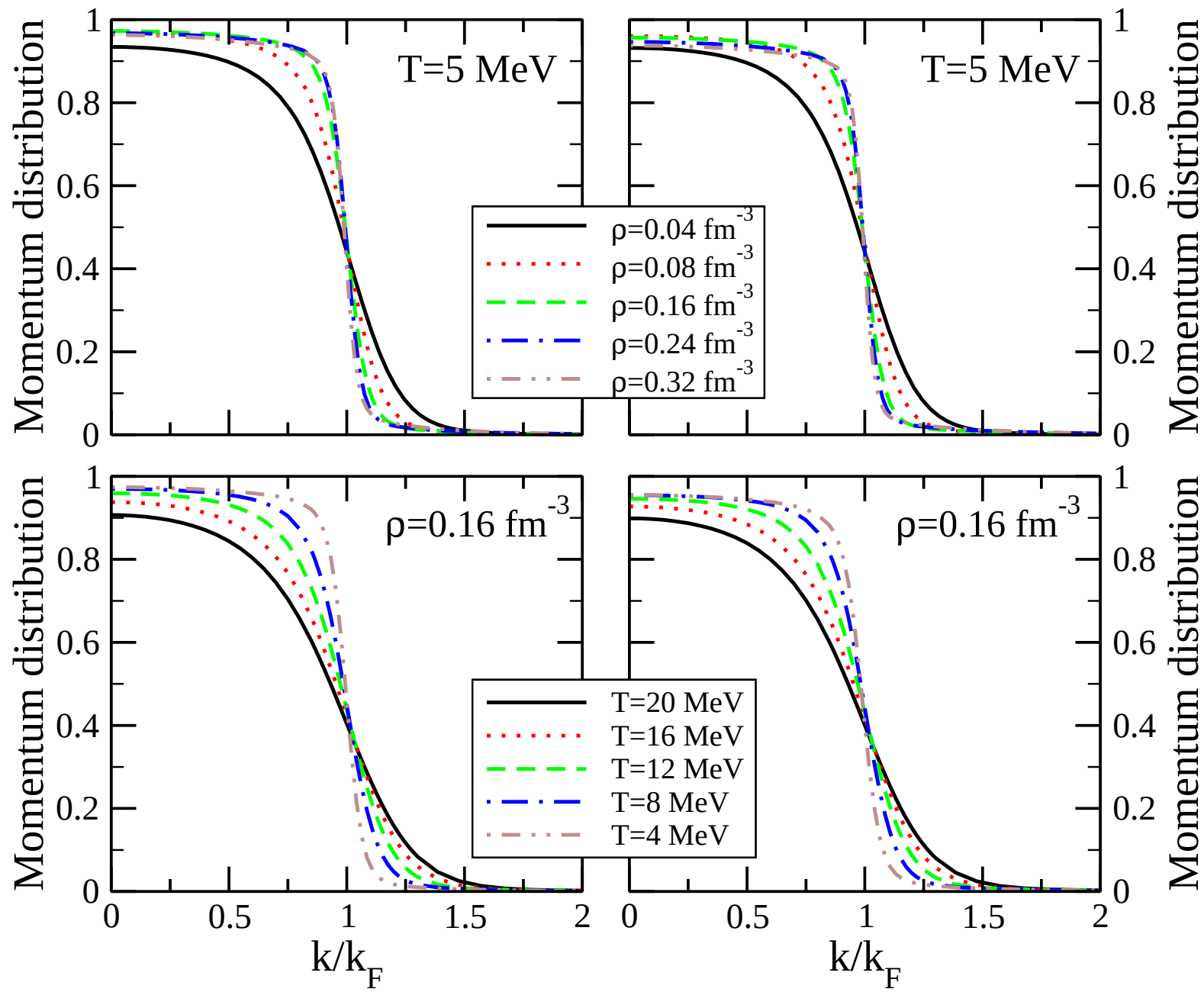


Real part of the on-shell self-energy for neutron matter

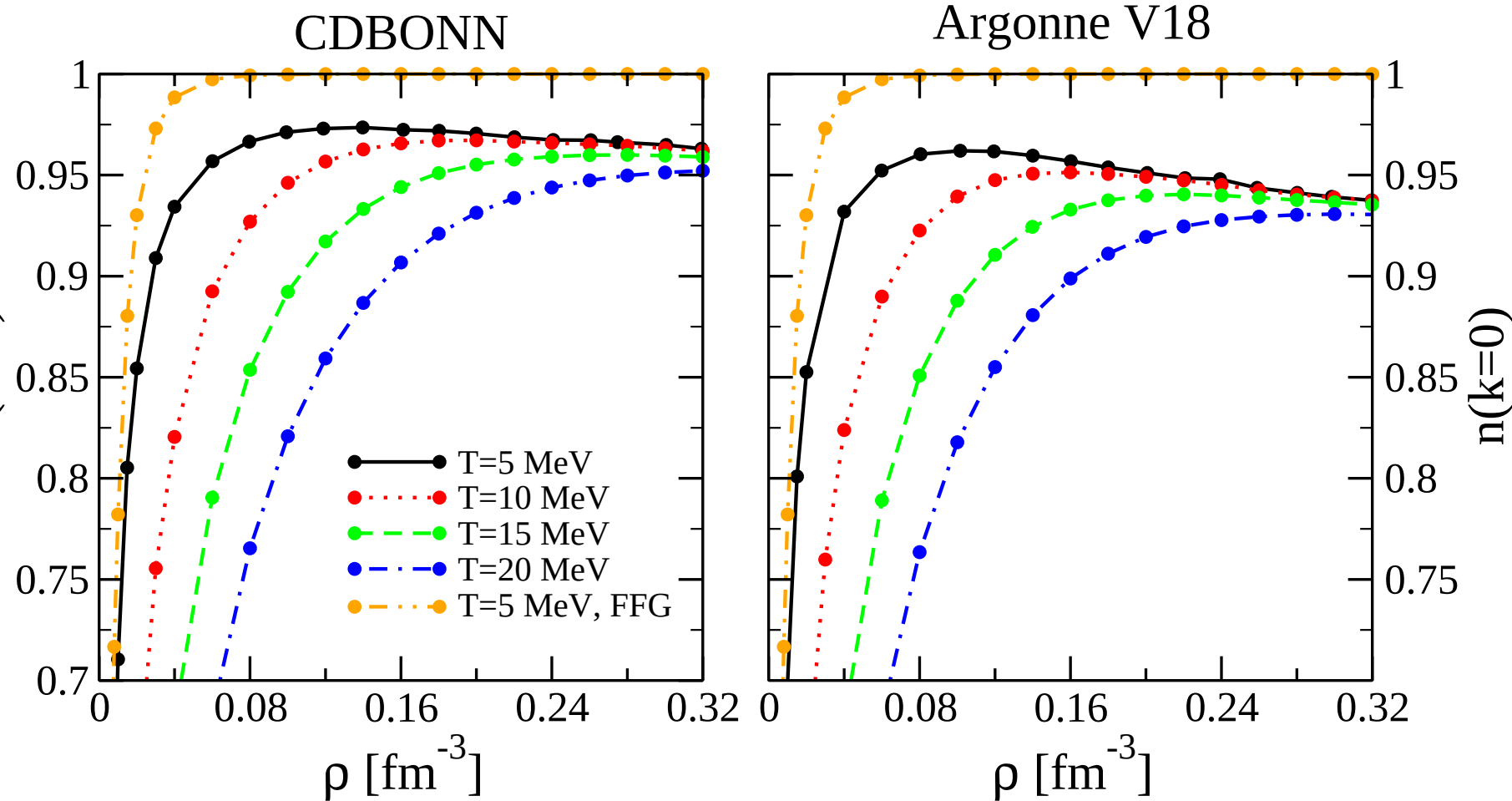


CDBONN

Argonne V18



Occupation of the lowest momentum state as a function of density for neutron matter



Summary

- ❖ The calculation and use of the single particle green function is suitable and it is easily extended to finite T . Temperature helps to avoid the “np” pairing instability.
- ❖ The propagation of holes and the use of the spectral functions in the intermediate states of the G -matrix produces repulsion. The effects increase with density.
- ❖ Important interplay between thermal and dynamical correlation effects.
- ❖ For a given temperature and decreasing density, the system approaches the classical limit and the depletion of $n(k)$ increases.
- ❖ For larger densities, closer to the degenerate limit, dynamical correlations play a more important role and $n(0)$ decreases with increasing density.
- ❖ For a given density when the asymmetry increases, the neutrons get more degenerate and the protons lose degeneracy. The depletion of the protons is larger and has important thermal effects.
- ❖ Three-body forces should not change the qualitative behavior.